Many engineering problems can be solved with relative ease on an analog computer. Typical use is the determination of design factors and operating characteristics without construction of a model. Here a computer is used to solve two design problems.

Putting the Analog Computer to Work

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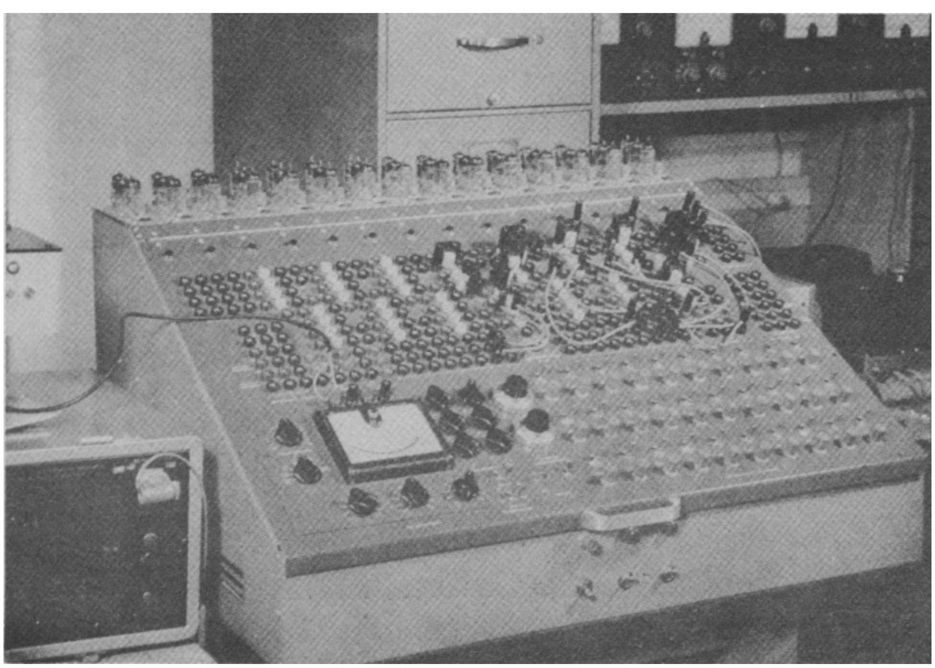
THE ANALOG COMPUTER has a direct application in many fields. There are thousands of problems in engineering and non-engineering fields which can be set up and solved with relative ease on a computer. Not only is time saved in the initial solution of the problem, but the constants can be changed, if necessary, without having to rework or setup the problem again.

The analog computer adapts itself particularly well to problems where the cost involved is too great to afford a failure. These problems can first be setup on the computer and tested to see if they are valid. In cases where failure occurs the cause can be determined and means taken to correct it.

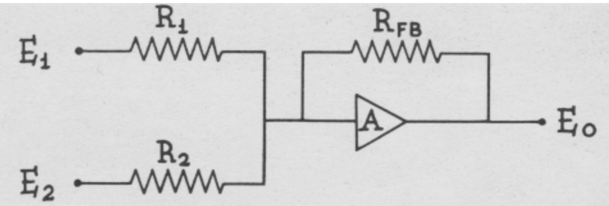
Solving problems on an analog computer consists of establishing relations between voltages and real time that are mathematically equivalent to the problem variables. Voltages are used as the dependent variables and time as the independent variable.

The equivalent relations for a particular differential equation are established through the use of computing elements which perform mathematical operations of addition, multiplication by a constant, and integration. Computer solution of an ordinary differential equation is thus accomplished by simulating the equation with these computing elements; setting the dependent variables (voltages)) to the correct initial conditions; and then placing the computer into operation which forces the voltages to vary in a manner prescribed by the differential equation. The voltage variations are recorded with respect to the independent variable (time). Recorders used are usually direct-inking pen recorders which plot a permanent record of the solution.

The computing elements are comprised mainly of high gain dc amplifiers with various feedback networks. The transfer functions of such devices are determined by these networks. Amplifiers employing resistive feedback networks constitute computing elements for summing and multiplying by a constant.

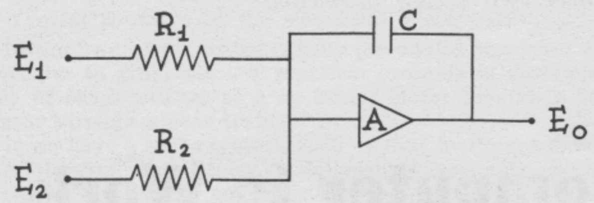


Heath desk-type computer plugged in for the mechanical bridge beam problem described in this article. The eight drawings of Figure 1, opposite page, illustrate the elements considered in the problem. Figure 2, page 272, is a block diagram of the computer wiring setup for problem.



$$E_o = \frac{R_{FB}}{R_1} E_1 + \frac{R_{FB}}{R_2} E_2$$

Amplifiers employing capacitive feedback networks constitute computing elements for integrating.



$$E_o = \frac{1}{R_1C} \int E_1 dt + \frac{1}{R_2C} \int E_2 dt$$

Relays are employed across integrating amplifiers to hold the amplifier output at the same potential as the input. In problems having initial conditions, a voltage source is placed in series with the relay. This charges the condenser across the amplifier to the initial condition potential required. When the computer is placed into operation these relays open, allowing the amplifiers to start integration.

A Typical Problem in the Mechanical Field

A practical problem would be that of finding the "bending moment" and "elastic curve diagrams" for the beams of a bridge that support a bed or roadway. This can be done by first generating the load diagram for the bridge and then finding the second and fourth integral respectively. The 8 parts of Figure 1 (a through h) are diagrammatic expressions of various aspects of this problem. Figure 2 shows the computer circuitry for this bridge beam problem.

Choosing arbitrary values for simplicity of the problem, suppose we wish to construct a bridge 30 ft. long and support it at the ends. Knowing the weight of the bed and beams and any other loads applied to the bridge, we can immediately setup the problem and draw the load diagrams, Figure la. In this case the load diagram is actually composed of three individual load diagrams: the beam and bed load diagram, Figure lb; the support diagram, Figure 1d; and the truck load diagram, Figure lc. Table 1 gives the values for the reactions appearing at F_1 and F_2, when the truck is placed at different locations on the bridge.

By placing the truck at some particular spot, we can generate the load diagrams. In the beam and bed load diagram, the load was assumed uniform and generated with a pulse width of 3 seconds. This gives a conversion factor of 1 ft. equals 0.1 seconds. The bridge supports are 6" wide. Therefore the pulse widths of the supports are 50 milliseconds. The loads applied from the wheels of the truck have a width of 6" each and are assumed to have a uniform load distribution. This also gives a pulse width of 50

Table 1

X	F_1	F_2
0 to 10 feet	F_1= 4,500—100X	F_2=1,500+100X
10 to 30 feet	F_1=12,500—300X	F_2= 300X—500
30 to 40 feet	F_1= 9,500—200X	F_2= 200X—500

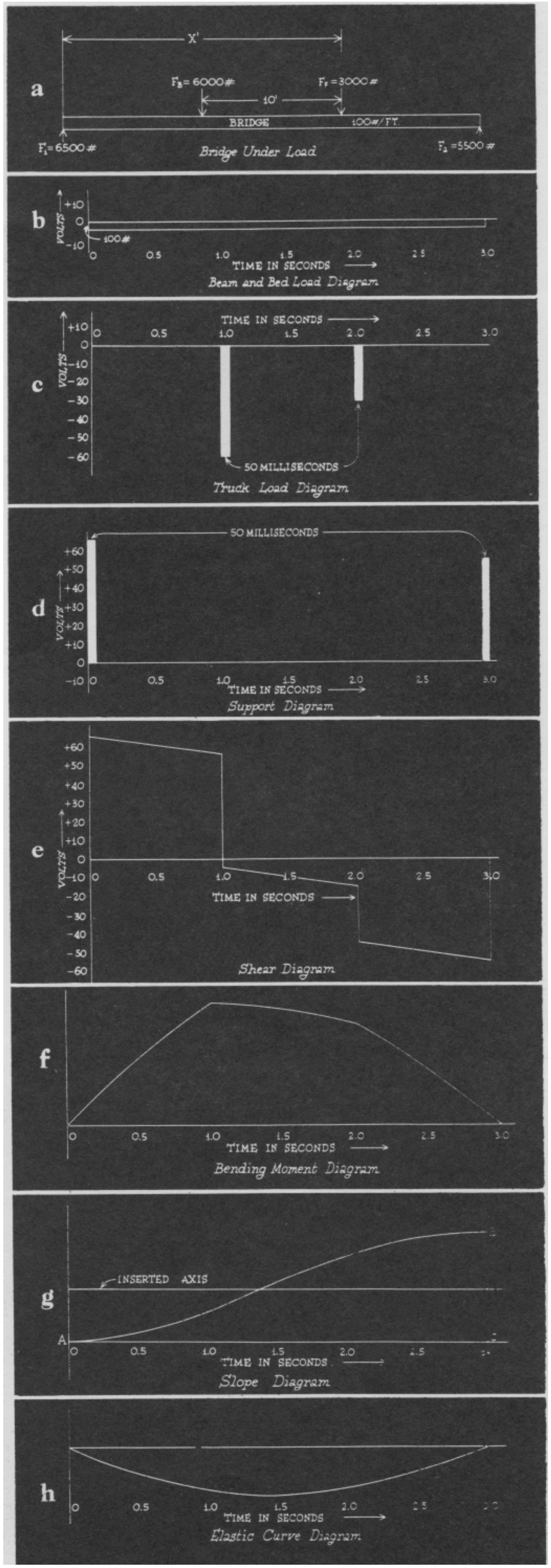


Figure 1, Right.

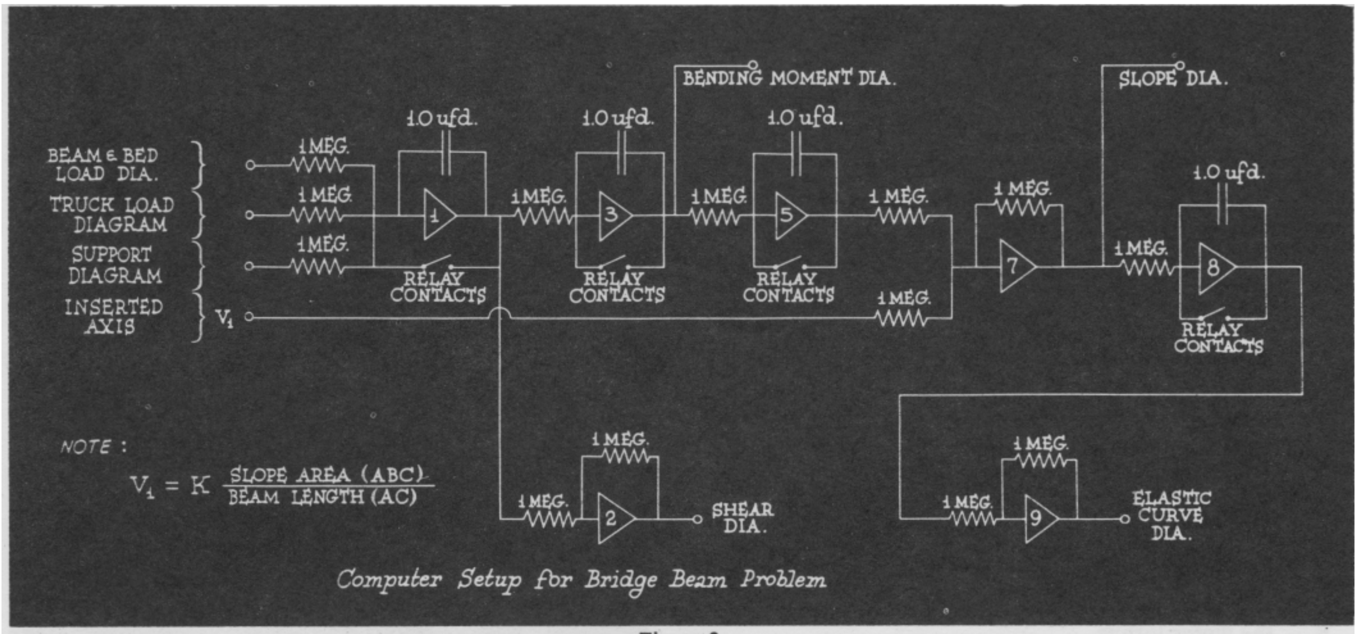


Figure 2.

milliseconds for the tires. These are shown in Figures lb, lc and ld, and have conversion factors of 1 volt equals 100 pounds and I second equals 10 ft. The illustration at bottom left of page 270, opposite, shows the Heath computer setup for this problem.

These three curves are generated individually on an external pulse generator and then fed into a combination summing and integrating amplifier, Figure 3. The output of this amplifier is the shear diagram, Figure le. This is fed to another integrator, the output of which is the bending moment diagram, Figure 1f. Feeding this to another integrator yields the slope diagram, Figure 1g. This is then fed to amplifier 7 which inserts a new axis in this slope diagram. This would normally be done by summing the slope curve with a constant negative dc voltage. This voltage is equal to the area under the slope curve (area ABC) divided by the length of the beam (length AC), and multiplied by the same scaling factor that the slop diagram was multiplied by.

Because the slope diagram at the output of amplifier 5 was negative, a positive dc voltage was used to represent the new axis. The two voltages are summed in amplifier 7, the output of which is the corrected slope diagram. This is then fed to the final integrator which yields the elastic curve diagram. This elastic curve is the shape or form to which the beams in the bridge are actually trying to conform. These three diagrams are shown in Figure 1e, 1f, 1g and 1h respectively.

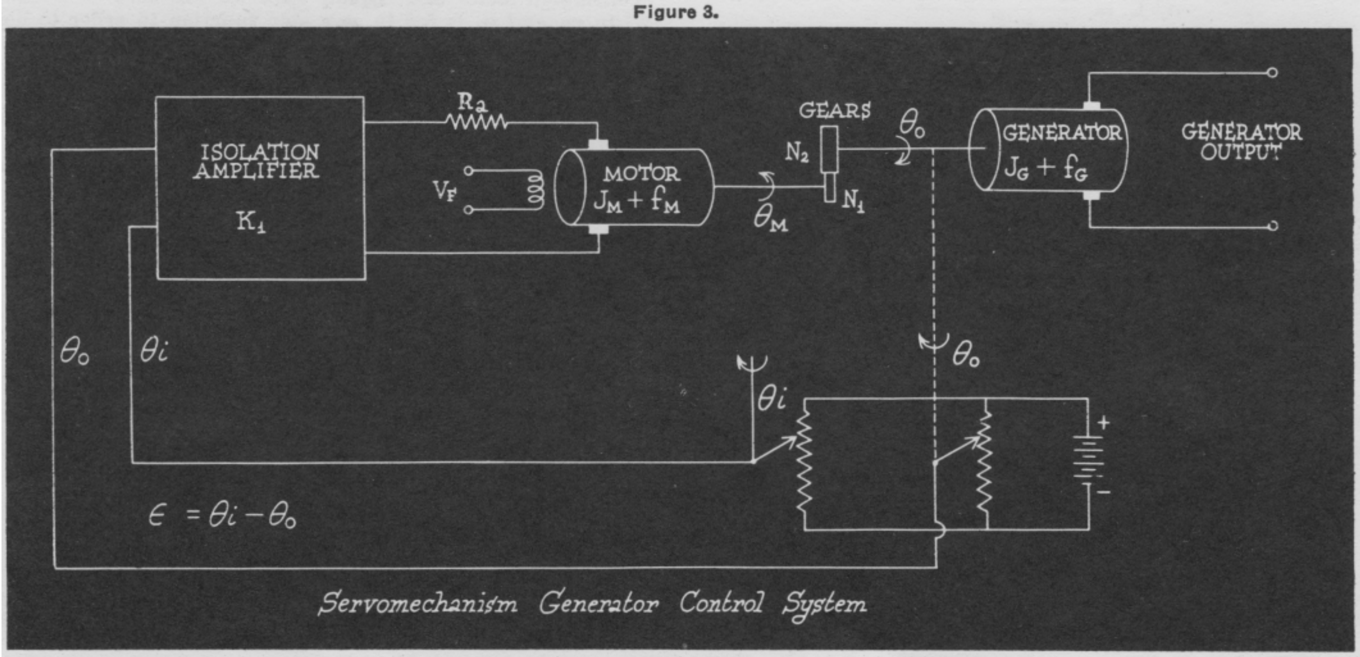
Because the amplifier outputs are 180° out of phase with the inputs, the shear diagram and the elastic curve diagram come out negative or 180° reversed from what they should be. To correct this they are fed respectively to sign changing amplifiers.

The integrating equation for an amplifier is

$$e_o = -\frac{1}{RC}\int e_{in}\,dt \text{ where } \frac{1}{RC}$$

is the time constant. Therefore by choosing different values for R and C we can run our problem at different time rates slower than real time, real time, or faster than real time. In the case of the above problem, R and C were 1 megohm and 1.0 ufd respectively. Using these values the time constant $\frac{1}{RC}$ becomes unity or real time.

Figure 3.



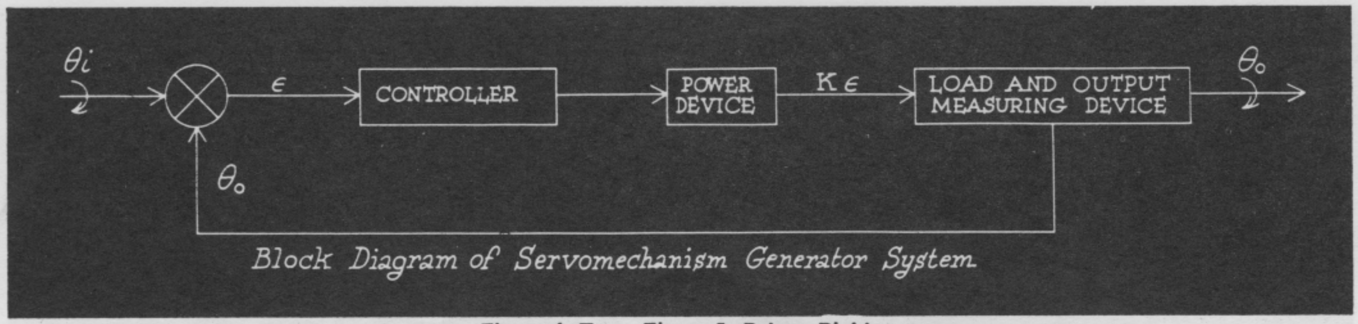


Figure 4, Top. Figure 5, Below, Right.

A Typical **Problem in the Electrical Field**

In the field of servomechanisms the analog computer has been put to great use. A practical example is determination of characteristic of a de Shunt Motor driving a generator through a gear train.

If we have a servo system, such as that in Figure 3, with the following experimentally determined constants, we can program this on the computer and find its operating characteristics without actual construction of the system.

K_t	the motor torque
K_m	the back emf
K_1	the isolation amplifier gain
J_m	the motor armature inertia
J_g	the generator inertia
f_m	the motor friction
f_g	the generator friction
N	the turns ratio
R_a	the motor armature resistance.

The motor has negligible armature inductance and is excited from a constant voltage source. Also the input and error detecting potentiometer have an effective 360° of mechanical travel.

First setup the block diagram as shown in Figure 4 and then write the equations for it.

The generator torque:

$$T_g = \frac{d^2\Theta_o}{dt^2}\left[J_g + \frac{J_m}{N^2}\right] + \frac{d\Theta_o}{dt}\left[f_g + \frac{f_m}{N^2}\right]$$

The motor torque:

$$T_m = N\,T_g = K_t\,i_a$$

The armature current

$$i_a = \frac{K_1\,E - K_m\,\frac{d\Theta_m}{dt}}{R_a}$$

The output displacement:

$$\Theta_o = N\,\Theta_m$$

Where:

$$N = \frac{N_1}{N_2}$$

$$E = \Theta_i - \Theta_o = \text{(Error)}$$

Substituting we arrive at:

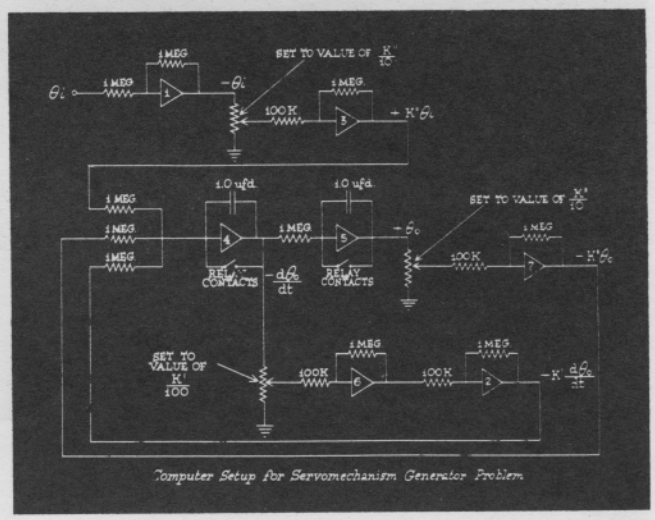
$$K_1\,(\Theta_i - \Theta_o) - \frac{K_m}{N}\frac{d\Theta_o}{dt} = \frac{R_a N}{K_t}\left[\frac{d^2\Theta_o}{dt^2}\left(J_g + \frac{J_m}{N^2}\right) + \frac{d\Theta_o}{dt}\left(f_g + \frac{f_m}{N^2}\right)\right]$$

Solving for the highest order differential of Θ_o:

$$\frac{d^2\Theta_o}{dt^2} = -\left[\frac{K_t\,K_m + R_a\,N^2\,f_g + R_a\,f_m}{R_a\,(N^2\,J_g + J_m)}\right]\frac{d\Theta}{dt} - \left[\frac{N\,K_t\,K_1}{R_a\,(N^2\,J_g + J_m)}\right]\Theta_o + \left[\frac{N\,K_t\,K_1}{R_a\,[N^2\,J_g + J_m]}\right]\Theta_i$$

Let:

$$K' = \left[\frac{K_T\,K_m + R_a\,N^2\,f_g + R_a\,f_m}{R_a\,(N^2\,J_g + J_m)}\right]$$



$$K'' = \left[\frac{N\,K_T\,K_1}{R_a\,(N^2\,J_g + J_m)}\right]$$

Then:

$$\frac{d^2\Theta_o}{dt^2} = -K'\frac{d\Theta_o}{dt} - K''\,\Theta_o + K''\,\Theta_i$$

This is now in computer form and is setup on the computer as shown in the block diagram of Figure 5. Because the constants K' and K" are usually both larger than unity, amplifiers have to be used in the feedback loop.

Oi is the forcing function and is measured in degrees. This is converted into volts and fed into the computer. In this particular problem Oi can vary from 0° to 360°, so a conversion factor of 1° equals 0.2 volts was used.

In general Oi is fed in as a step function in order to determine the correctiveness and accuracy of the servo system.

Although these are relatively basic problems, problems of much higher degree and of a more complicated nature can be solved with less time and expense than would be involved if the problem were actually constructed and then tested.

MEET THE AUTHOR

CARL HEALD is a Project Engineer at Heath Co., Benton Harbor, Mich., a subsidiary of Daystrom, Inc. He is currently working on the analog computer project at the Heath Co. Mr. Heald graduated in 1955 with an Electrical Engineering Degree from Michigan College of Mining and Technology. Prior to joining the Heath Co. he was associated with the Vacuum Tube Research Division of Wright-Patterson Aircraft, Dayton, Ohio.